Coin Cradle pendulum, v2: predicted from the hardware, with nothing fitted

Buyandsellbullion.com technical report

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This version replaces the fitted gap and magnet of v1 (technical report v1, 25 September 2026, unchanged) with the measured hardware. Labels: [measured], [derived], [assumed], [memory], [fitted].

Note added 26 September 2026. The outward-swing ratchet rule that the data selected in Section 1 was later confirmed on the device on 26 September 2026.

Summary

- **Free parameters:** none continuous. The one unknown left is how the dial is driven, which no document describes. Of three ratchet rules, the data select "the dial advances only on outward strokes on the far side". That is one discrete choice.
- **Error with nothing fitted:** 20.9% rms in $\ln R$ over the 38 readings with a known alloy (21.2% on v1's 31); the model reads 16% high on average, and 68% of readings fall within ±25%.
- **With one fitted friction torque,** $\tau_f = 1.5 \times 10^{-4}\,\mathrm{N\,m}$ (19% of the spec bound): 14.7% rms, and 14.7% leave-one-out (13.7% on the 31). v1 needed three fitted numbers for 13.8%.
- **Moment of inertia:** $I = 1.238,\ 1.180,\ 1.124 \times 10^{-3}\,\mathrm{kg\,m^2}$ at $S = 1, 2, 3$. The magnet contributes 82%, the rod 18%, and the axle and screw 0.1%.

1 Inputs

quantity	value	status
magnet	N52 ring, axial; OD 23.4, bore 6.1, height 20 mm; 60.4 g	measured
B_r	1.42–1.47 T (central 1.445)	grade range
face to deck	exactly $3S$ mm (gauge-block spacers)	measured
pivot height	142 mm above the deck	measured
pivot to magnet centre	$142 - 3S - 10\,\mathrm{mm} = 129, 126, 123\,\mathrm{mm}$	derived
release	50°	measured
rod	48.2 g, uniform from pivot to magnet top	mass measured; shape assumed
cross axle	32.7 g solid cylinder, radius 4 mm	radius assumed
side screw	4.27 g at 14 mm; no gravity moment	orientation assumed
coin	centred; rests on its rims; metal slab $t_{\mathrm{eff}} = m/\rho\pi R^2$ centred at half the rim height	derived / rims from JSON or memory
σ	workbook *Coin Types*	as v1

$$I = \tfrac{1}{3}m_{\mathrm{rod}}L_r^2 + m_{\mathrm{mag}}\Big[\tfrac{R_o^2+R_i^2}{4} + \tfrac{H^2}{12} + d_m^2\Big] + \tfrac{1}{2}m_{\mathrm{ax}}r_{\mathrm{ax}}^2 + m_{\mathrm{sc}}r_{\mathrm{sc}}^2, \qquad Mg\ell = g\big(m_{\mathrm{rod}}L_r/2 + m_{\mathrm{mag}}d_m\big).$$

Here $d_m = 142 - 3S - 10$ mm and $L_r = d_m - 10$ mm. The period is 0.68, 0.68 and 0.67 s; the release energy $Mg\ell(1 - \cos 50°)$ is 37.4, 36.5 and 35.6 mJ.

2 Physics

Exact ring field [derived]. An axially magnetised ring is equivalent to surface charge $\pm B_r/\mu_0$ on its two annular faces. At depth h below the near face,

$$B_z(\rho, h) = \frac{B_r}{2} \int_0^\infty [R_o J_1(kR_o) - R_i J_1(kR_i)]\, J_0(k\rho)(e^{-kh} - e^{-k(h+H)})\, dk.$$

On the axis this reduces to $\frac{B_r}{2}[f(h+H, R_o) - f(h, R_o) - f(h+H, R_i) + f(h, R_i)]$ with $f(z, a) = z/\sqrt{z^2 + a^2}$; the quadrature matches it to 2×10^{-5}. The on-axis field is 0.255 T at 3 mm and 0.176 T at 10 mm, and dips to 0.08 T at 1 mm because of the bore.

Absolute braking [derived]. As in v1, $\nabla^2\psi = \partial_x B_z$ in each lamina, with $\psi = 0$ on the coin's edge, and

$$c(x, h) = \sigma\, B_r^2 \sum_{\text{depth}} \int |\nabla\psi|^2 dA \quad [\text{N s/m}],$$

with no free scale. Check: for a dipole over an infinite sheet this gives $\frac{1}{2}\sigma t v \int B_z^2 dA = 3\mu_0^2 m^2 \sigma t v/(128\pi z^4)$, which is Reitz's low-speed drag.

Equation of motion [derived].

$$I\ddot{\theta} = -Mg\ell \sin\theta - c(x, h) R_f^2 \cos^2\theta\, \dot{\theta} - \tau_f \operatorname{sgn}\dot{\theta}, \qquad x = R_f \sin\theta, \quad h = g_S + R_f(1 - \cos\theta) + \zeta,$$

where $g_S = 3S - \frac{1}{2}(t_{\text{rim}} + t_{\text{eff}})$ is the gap to the top of the metal slab. The rise of the magnet off the coin along the arc is now fixed by the geometry. The equation is integrated with RK4 at 0.4 ms, which agrees with 0.15 ms to 10^{-5}. The braking is light beside gravity ($D/I \le 7\,\text{s}^{-1}$, $\omega_0 \approx 9.3\,\text{s}^{-1}$), so the energy budget (release energy against the loss per pass, $\int c\, v\, dx$) sets how many swings occur.

3 What the dial reads

The spec records cumulative degrees including full turns; the dial is marked "TURN CLOCKWISE ONLY"; a forum user reports that the dial "kept going"; and the site's dial animation is cosmetic. The dial is therefore ratcheted and cumulative, but its drive rule is undocumented. Three rules were tested, each with a one-to-one drive and nothing fitted:

dial counts	rms	mean $\ln(\hat{R}/R)$	spread
all travel in the return direction	0.74	+0.71 (2.0× high)	0.18
outward strokes, far side only	**0.21**	**+0.16**	**0.14**
outward strokes, release side only	0.33	−0.16	0.29

Under the far-side rule, the reading is about the sum of the amplitudes on the side away from the release. Return travel behind a 2:1 reduction gives the same level (best scale 0.49) but a worse shape. **Test:** move the pendulum slowly by hand on each side and watch the dial. Resting the coin on its rims, rather than laying it flat as a slab on the deck, improves the rms from 0.29 to 0.21.

4 Prediction against the readings

Of the 44 readings, 38 are predicted. The six 40% clad coins have no book conductivity; the model infers $\sigma = 48$–65 MS/m for them. With nothing fitted the rms is 20.9%. The largest errors are on the 1/10 oz coins, which read 37–55% high; they take the longest to stop, which is the

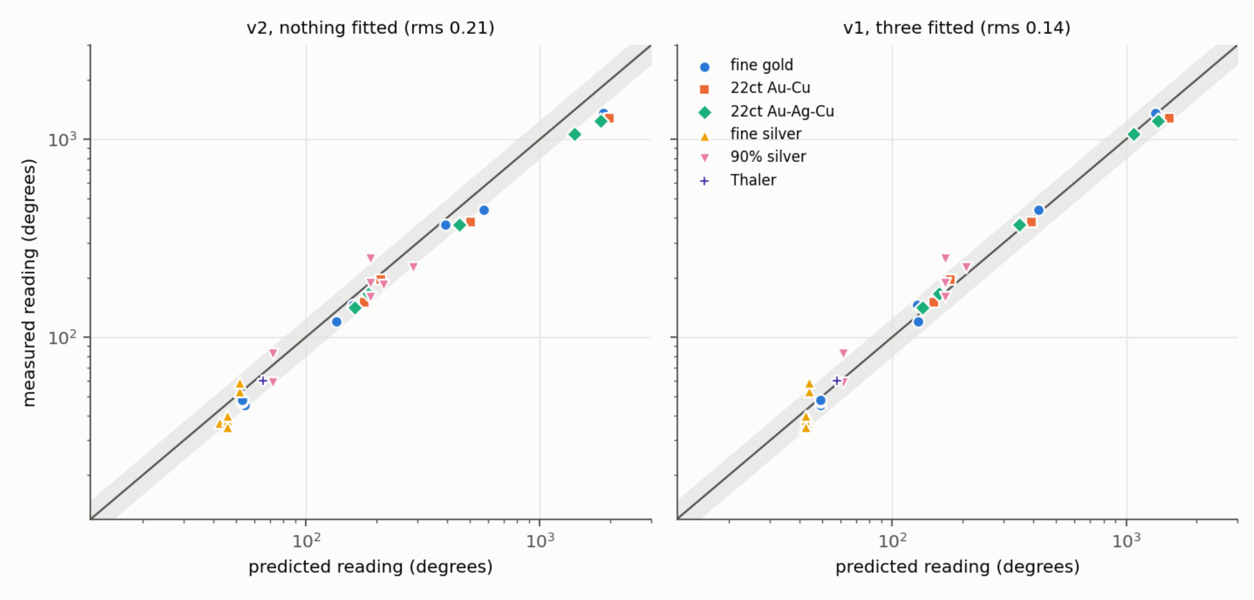

Figure 1: Measured against predicted readings. Left: v2, nothing fitted. Right: v1, three fitted. Grey band: ±25%.

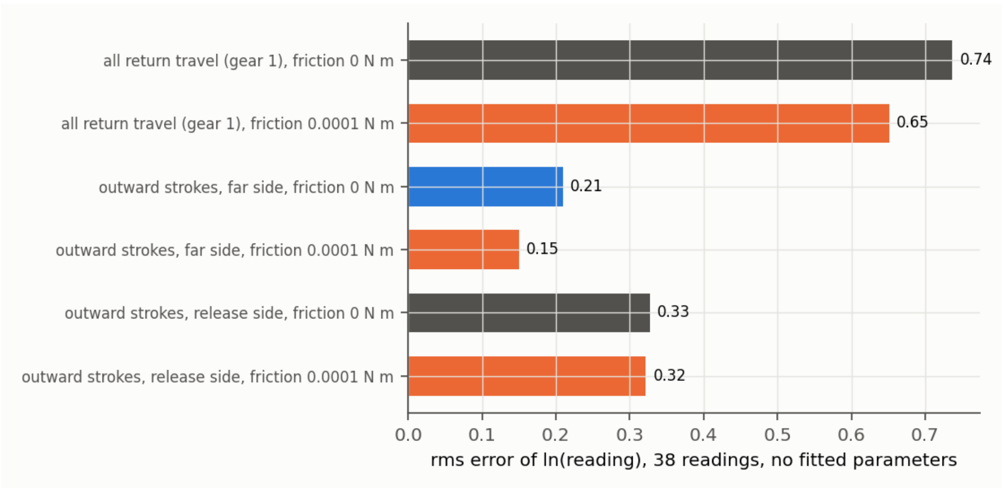

Figure 2: Zero-parameter error under each dial rule, with and without friction.

signature of the unmodelled friction. Fitting that one torque gives 14.7% (leave-one-out 14.7%), with $\tau_f = 1.5 \times 10^{-4}\,\mathrm{N\,m}$ against the spec bound of 8.0×10^{-4}.

Uncertainty (nothing refitted):

- B_r from 1.42 to 1.47 T: ∓3.5% on every reading.
- Rod inertia ±15%: ±1.4%.
- Axle radius 2–6 mm: under 0.2%.
- B_r and inertia drawn jointly: 1.8% per reading, with rms 0.19–0.23 across draws.
- Friction at the spec bound: −8% to −75%.

The hardware uncertainty is an order of magnitude below the residual.

5 Comparison with v1

	v1	v2
fitted	3 ($A, z_1, \Delta z$)	0 continuous + 1 discrete; or 1 (friction)
error, 31 readings	13.8% (LOO)	21.2% unfitted; 13.7% (LOO) with friction
spacer	3.36 mm (3.11–3.59)	3.0 exact
gap at $S = 1$	$z_1 = 18.7\,\mathrm{mm}$	3.0 mm to face, 13 mm to centre

v1's spacer figure is 12% high, with 3.0 just outside its interval: its invented pot-magnet field fell off at the wrong rate, and the fitted step compensated for that. z_1 cannot be reconciled with either the face (3 mm) or the centre (13 mm). It was the distance at which the invented magnet gave about the right field shape, and v1 had already flagged it as unidentified (the cylinder variant gave 8.2 mm). v1's A absorbed B_r, I, the release energy and the dial drive. The discrimination conclusions of v1 still hold, because for a fixed outline the reading scales as $1/(\sigma t)$.

6 What to measure next

1. The dial drive: turn the pendulum slowly by hand on each side and note when the dial advances. (Since done: the outward-swing rule was confirmed on 26 September 2026.)
2. A free decay from 50° with no coin, to measure τ_f directly.
3. Ten releases per coin, including the plated-W Eagle.
4. Rim thicknesses with callipers, and the drawings (their effect on I is only ±1.4%).